Άσκηση 1η

Να βρείτε όλες τις μερικές παραγώγους δευτέρας τάξεως των παρακάτω συναρτήσεων:

i) $f(x,y) = xy + (x+2y)^2$, ii) $f(x,y) = ημx \cdot ημ^2 y$

iii) $f(x,y) = x \cdot e^y + yx^2$, iv) $f(x,y) = e^{-x^2-y^2}$.

Τι παρατηρείτε στην (iii);;

ΛΥΣΗ

i) • $\dfrac{\partial f}{\partial x}(x,y) = y + 2(x+2y)\dfrac{\partial}{\partial x}(x+2y) = 5y + 2x$

$\dfrac{\partial^2 f}{\partial x}(x,y) = 2$, Άρα, $\dfrac{\partial^2 f}{\partial x^2}(x,y) = 2$

• $\dfrac{\partial f}{\partial y}(x,y) = x + 2(x+2y)\dfrac{\partial}{\partial y}(x+2y) = 5x + 8y$

$\dfrac{\partial^2 f}{\partial y}(x,y) = 8$, Άρα $\dfrac{\partial^2 f}{\partial y^2}(x,y) = 8$

• $\dfrac{\partial^2 f}{\partial x \partial y}(x,y) = \dfrac{\partial f}{\partial x}\left(\dfrac{\partial f}{\partial y}(x,y)\right) = \dfrac{\partial f}{\partial x}(5x+8y) = 5$

• $\dfrac{\partial^2 f}{\partial y \partial x}(x,y) = \dfrac{\partial f}{\partial y}\left(\dfrac{\partial f}{\partial x}(x,y)\right) = \dfrac{\partial f}{\partial y}(5y+2x) = 5.$

ii) $\dfrac{\partial f}{\partial x}(x,y) = συνx \cdot ημ^2 y$, $\dfrac{\partial f}{\partial y}(x,y) = (2 \cdot ημy \cdot συνy)\,ημx =$

$\dfrac{\partial^2 f}{\partial x}(x,y) = -ημx \cdot ημ^2 y$, $= ημx \cdot ημ2x$

$\dfrac{\partial^2 f}{\partial y}(x,y) = 2 \cdot ημx \cdot συν2y.$

$\dfrac{\partial^2 f}{\partial y \partial x}(x,y) = 2συνx \cdot ημy \cdot συνy = συνx \cdot ημ2x$

$\dfrac{\partial^2 f}{\partial x \partial y}(x,y) = συνx \cdot ημ2y.$

iii) • $\frac{\partial f}{\partial x}(x,y) = e^{y} + 2xy$, $\frac{\partial f}{\partial y}(x,y) = x \cdot e^{y} + x^2$

• $\frac{\partial^2 f}{\partial x^2}(x,y) = 2y$, $\frac{\partial^2 f}{\partial y^2}(x,y) = x \cdot e^{y}$

• $\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}(x,y)\right) = \frac{\partial f}{\partial x} \cdot (x \cdot e^{y} + x^2) = e^{y} + 2x$

• $\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}(x,y)\right) = \frac{\partial f}{\partial y}(e^{y} + 2xy) = e^{y} + 2x$

Βλέπουμε ότι $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

iv) • $\frac{\partial f}{\partial x}(x,y) = e^{-x^2-y^2} \cdot \frac{\partial}{\partial x}(-x^2-y^2) = e^{-x^2-y^2}(-2x)$

$\frac{\partial^2 f}{\partial x^2}(x,y) = \frac{\partial}{\partial x}\left(-2e^{-x^2-y^2} \cdot x\right) =$

$= -2\left[e^{-x^2-y^2} \cdot (-2x) \cdot x + e^{-x^2-y^2}\right] =$

$= 4x^2 e^{-x^2-y^2} - 2e^{-x^2-y^2} =$

$= e^{-x^2-y^2} \cdot \left[4x^2 - 2\right]$

• $\frac{\partial f}{\partial y}(x,y) = -2y \cdot e^{-x^2-y^2}$

$\frac{\partial^2 f}{\partial y^2}(x,y) = e^{-x^2-y^2} \cdot \left[4y^2 - 2\right]$

Ενώ $\frac{\partial f}{\partial x \partial y}(x,y) = \frac{\partial f}{\partial y \partial x}(x,y) = 4xy \cdot e^{-x^2-y^2}$